International Journal of Advanced Science and Technology
Vol. 29, No. 6s, (2020), pp. 1770-1777

Time Truncated Modified Group Chain Sampling Plans for Marshall Olkin Extended Lomax Distribution

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Abstract

Time truncated life test and group sampling technique are introduced for time saving purpose by limiting time of inspection and providing a platform for multiple inspection respectively. The issue of rapid change in operating characteristics faced by sampling plan with zero acceptance number was fixed by chain sampling plan which considers preceding lots in the acceptance criteria. Modified chain sampling reworks chain sampling by providing tighter inspection in order to give more protection to consumer. In this article, the acceptance sampling plans for attributes called Modified Group Chain Sampling Plan (MGChSP) for Marshall Olkin Extended Lomax (MOEL) distribution is developed by combining group sampling and modified chain sampling technique. The performance of MGChSP is measured by obtaining minimum number of groups and probability of lot acceptance following selected values of design parameters.

Keywords: *Marshall Olkin Extended Lomax distribution, Number of Groups, Probability of Lot Acceptance, Truncated life test.*

1. Introduction

The high value of product's lifetime would consume time in waiting until a defective item is found during inspection. The idea of time truncated life test is to observe the number of defective items until a pre-assigned termination time, t_0. If the number of defective items observed, d is at most equal to a pre-specified acceptance number, c during the termination time, t_0 , then the lot will be accepted. Otherwise, the lot will be rejected. It is also convenient to assign the termination time t_0 as a multiple of the specified mean life, $a\mu_0$ where a is a constant.

In quality control where 100% inspection will contribute to a very high cost, single sampling plan (SSP) is most commonly used due to its simplicity in application. This plan is defined by three design parameters: lot size, N; sample size, n; and acceptance number, c. It considers only one random sample from the whole lot with only one acceptance number. SSP has been studied by many researchers such as [1] and [2] who proposed SSP based on truncated life test by considering Weibull and normal, and log normal as lifetime distributions respectively.

Various types of sampling plan have been developed by many researchers from time to time in order to provide more options to industries in assuring their quality of products. Previously, SSP with acceptance number equal to zero has been developed by providing a very tight inspection in order to give more protection to the consumer. Unfortunately, it has an obvious shortcoming where the probability of lot acceptance, $L(p)$ drops at a very fast rate corresponding to the small increase of proportion defective, which is rather unfair to the producer. Consequently, chain sampling plan (ChSP-1) was introduced by [3] to overcome the problem. In ChSP-1, the cumulative data of more than one sample is taken into account. The decision of accepting or rejecting the current lot in this case is based on the number of defective items found in the current inspected sample and the quality of the preceding lots. The current inspected lot will be accepted if there is no defective items found in

its sample. The lot can also be accepted if it has only one defective item provided that the preceding lots must be free from any defectives.

Motivationally, ChSP-1 provides a second chance to the producer by considering preceding lots as an extra condition of lot sentencing. Statistically, more information (data) provides better estimation of the parameter of interest, hence resulting in more accurate inference. Based on the comparison between ChSP-1 and SSP with zero acceptance number provided by [4], it is shown that ChSP-1 increases the $L(p)$ at high levels of quality, but maintains the $L(p)$ at low levels of quality. Later, [4] introduced modified chain sampling plan (MChSP-1), which safeguards consumer better than previous ChSP-1. By taking SSP with zero acceptance number as a baseline, MChSP-1 reduces the $L(p)$ at low levels of quality which is beneficial for the consumer, while at the same time keeping the $L(p)$ high for the product with high level of quality, which preserves the protection to the producer. In addition, it is found that MChSP-1 produces smaller sample size if compared to ChSP-1, which means MChSP-1 is better in cost saving.

In industrial application, there is an urgency of having a simultaneously testing multiple items as to save the time and cost of sampling inspection. As a result, group sampling plan (GSP) has been developed. The sample size is the main concern in the development of GSP because time and cost of the inspection are related to the number of inspected items, which is determined by the sample size. The items in the sample are distributed into groups with equal size and all groups will be inspected simultaneously. The information of defective items found in every group is taken into account to proceed with the lot sentencing. GSP based on time truncated life test have been widely studied in recent time. For example, [5] developed a GSP under truncated life test based on inverse Rayleigh and log-logistic distribution. Later, [6] came out with a GSP by using generalized exponential distribution.

In 2015, [7] proposed a sampling plan by combining the features of GSP and ChSP-1 and it is called group chain sampling plan (GChSP). This plan is developed for Pareto distribution of the 2nd kind. In consequence, the benefits of both GSP and ChSP-1 have successfully been gathered in this new sampling plan which are saving in inspection time and boosting the $L(p)$ at good quality level (compared to SSP with zero acceptance number).

Encouraged by the approach of integrating more than one technique to have a better sampling plan, this study is intended to gather the advantages of both GSP and MChSP-1 in a sampling plan named as modified group chain sampling plans (MGChSP). The advantages are saving in inspection time, reducing the $L(p)$ at poor quality level while maintaining the $L(p)$ at good quality level (compared to SSP with zero acceptance number) and saving more in terms of inspection cost (compared to ChSp-1). MGChSP has been introduced by [8] for Pareto distribution of the second kind but has not yet been developed for any other distributions in literature. Therefore, this study proposes to develop modified group chain sampling plans (MGChSP), for time truncated life test following MOEL distribution.

2. Glossary of Symbols

g	:	Number of groups
r	:	Group size
n	:	Sample size
d	:	Number of defective items
c	:	Acceptance number
α	:	Producer's risk (Probability of rejecting a good lot)
β	:	Consumer's risk (Probability of accepting a bad lot)
$L(p)$	:	Probability of lot acceptance
p	:	Proportion defective
δ	:	Scale parameter
$\frac{\mu}{\mu_0}$	:	Mean ratio

a : Time termination multiplier

i : Number of preceding lots

3. Marshall Olkin Extended Lomax Distribution

The CDF is given by

$$F(t;\delta) = \frac{\left(1+\frac{t}{\delta}\right)^{\theta}-1}{\left(1+\frac{t}{\delta}\right)^{\theta}-\overline{v}}; \qquad (1)$$

$t > 0, \delta, v, \theta > 0, \quad \overline{v} = 1 - v,$

where δ, v, θ are the scale parameter, index parameter and shape parameter respectively.

It is fixed that $v = 2 \; and \; \theta = 2$ for the purpose of comparison with the existing sampling techniques in literature. [9] considered MOEL distribution with $v = 2$ and $\theta = 2$ for their proposed single sampling plans. More recently, [10] studied group sampling plan by using the same consideration for his group sampling plan.

The true mean life is given by $\mu = 1.570796\,\delta$ when $v = 2$ and $\theta = 2$. Then, proportion defective is given by

$$p = F(a\mu_0\,;\,\delta) = \frac{\left(1+\frac{a\mu_0}{\mu/1.570796}\right)^2-1}{\left(1+\frac{a\mu_0}{\mu/1.570796}\right)^2-(-1)} = \frac{\left(1+\frac{1.570796a}{\left(\frac{\mu}{\mu_0}\right)}\right)^2-1}{\left(1+\frac{1.570796a}{\left(\frac{\mu}{\mu_0}\right)}\right)^2+1}. \qquad (2)$$

4. Developing the Operating Procedure

The following Figure 1 shows the operating procedure of a MGChSP based on time truncated life test.

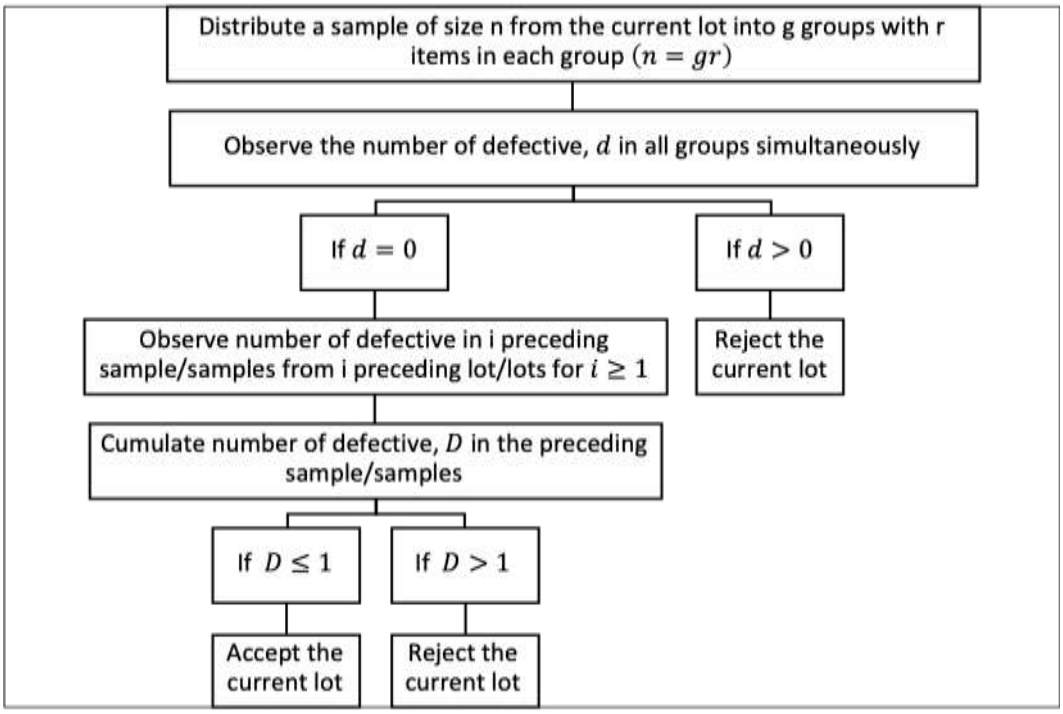

Figure 1. The operating procedure of MGChSP

The products with true mean life, μ greater than specified mean life, μ_0 are considered as 'good' product. The proposed plans are designed to meet that requirement where the lot will be accepted only if $\mu \geq \mu_0$. Otherwise, the lot will be rejected. By referring to the operating procedure of MChSP-1 which was developed by [4] and applying the idea of group sampling plan, proposed plans in this study (MGChSP) are constructed as the operating procedure shown in Figure 1. The operating procedure is detailed out in the following steps:

1. For each of the submitted lots, find the minimal number of groups, g and allocate r items to each group such that the sample size $n = gr$.
2. Specify the termination time, t_0.
3. Inspect all the groups simultaneously and record the number of defective items, d found in each of the groups. Terminate the inspection at $t = t_0$.
4. For the case of more than one preceding sample ($i > 1$), accept the lot if no defective items are found in the current under inspection sample (in all groups) provided the preceding i samples from i preceding lots also contain no defective items. Also, accept the lot if no defective items are found in the current under inspection sample on condition that any one of i preceding samples contains at most one defective item and the rest $(i - 1)$ samples have no defective items. Otherwise, reject the lot.

5. For the case of one preceding sample ($i = 1$), accept the lot if no defectives are found in the current lot provided that preceding sample has at most one defective. Otherwise, reject the lot.

The value of g must be at least 2 ($g > 1$). If $g = 1$, then the sampling plan will reduce to the basic modified chain sampling (MChSP-1).

5. Determining the Probability of Lot Acceptance

The probability of lot acceptance for the modified group chain sampling plan can be generalized as

$$P_a(p) = P_{0,n}\left[{P_{0,n}}^{i} + i\ {P_{0,n}}^{i-1} P_{1,n}\right], \quad (3)$$

where $P_{0,n}$ represents the probability of having zero defective item in a sample, while $P_{1,n}$ is the probability of having one defective item in the same sample. As we are dealing with success and failure, Binomial distribution is used to obtain the probability of having zero or one defective item in the lot. Then we can rewrite Equation (3) as follows:

$$P_a(p) = (1-p)^{gr}\left[(1-p)^{gri} + i\,(1-p)^{gr(i-1)}\, grp(1-p)^{gr-1}\right] \quad (4)$$

6. Analysis and Result

The chances of a product to be defective or commonly known as proportion defective, p is a compulsory element in obtaining minimum g and $L(p)$. Six different values of a at seven different levels of $\frac{\mu}{\mu_0}$ are used to obtain the values of p based on the formulas in Equation 2. Generated values of p when $a = 0.7, 0.8, 1, 1.2, 1.5, 2$ at $\frac{\mu}{\mu_0} = 1, 2, 4, 6, 8, 10, 12$ are presented in Table 1.

Table 1. Proportion defective for the product following MOEL distributions

a	$\frac{\mu}{\mu_0}$						
	1	2	4	6	8	10	12
0.7	0.6302	0.4121	0.2382	0.1667	0.1281	0.1039	0.0874
0.8	0.6717	0.4523	0.2666	0.1879	0.1449	0.1178	0.0993
1.0	0.7372	0.5224	0.3196	0.2284	0.1774	0.1449	0.1224
1.2	0.7855	0.5810	0.3680	0.2666	0.2085	0.1710	0.1449
1.5	0.8369	0.6518	0.4326	0.3196	0.2526	0.2085	0.1774
2.0	0.8898	0.7372	0.5224	0.3978	0.3196	0.2666	0.2284

Next, by referring to Equation 4 and by satisfying the inequality $L(p_0) \leq \beta$ in order to protect specified consumer's risk, the following inequality is used to obtain the minimum g:

$$(1-p)^{gr}\left[(1-p)^{gri} + i\,(1-p)^{gr(i-1)}\, grp(1-p)^{gr-1}\right] \leq \beta \quad (5)$$

Minimum g of MChSP-1 is then determined by using various values of p generated in Table 1. There are three common behaviors of minimum g corresponding to various change of design parameters ($i, \beta, \frac{\mu}{\mu_0}, r$ and a). These behaviors are related to pre-specified consumer's risk, β, preceding lots, i and time termination multiplier, a. Of our main interests are how these factors can reduce the minimum g since lower minimum g leads to lower sample size which is good for inspection cost saving.

The effect of pre-specified consumer's risk towards minimum g can be determined by observing the table of result (e.g. Table 2) vertically upwards (β increasing from 0.01 to 0.25). Generated

minimum g when $i = 1, r = 2, a = 0.7, \beta = 0.01, 0.05, 0.1, 0.25$ are put in bold to highlight the case in Table 2. For example, minimum g when lifetime follows MOEL distribution in Table 2 decreases from 2 to 1 as pre-specified consumer's risk increases from 0.01 to 0.25 by considering the pre-specified values of i, r and a.

Table 2. Minimum number of groups for MGChSP based on MOEL distribution with *i*=1

β	r	a					
		0.7	**0.8**	**1.0**	**1.2**	**1.5**	**2.0**
0.25	2	**1**	1	1	1	1	1
	3	1	1	1	1	1	1
	4	1	1	1	1	1	1
	5	1	1	1	1	1	1
0.10	2	**1**	1	1	1	1	1
	3	1	1	1	1	1	1
	4	1	1	1	1	1	1
	5	1	1	1	1	1	1
0.05	2	**2**	2	1	1	1	1
	3	1	1	1	1	1	1
	4	1	1	1	1	1	1
	5	1	1	1	1	1	1
0.01	2	**2**	2	2	2	1	1
	3	2	1	1	1	1	1
	4	1	1	1	1	1	1
	5	1	1	1	1	1	1

Next, the probability of lot acceptance, $L(p)$ is obtained by substituting the selected values of minimum g generated in Table 2 into the equation of $L(p)$ (Equation 4). Table 3 presents the $L(p)$ for seven levels of mean ratio and four levels of pre-specified consumer's risk based on $i = 1, r = 2$ as control design parameters.

Table 3. Probability of lot acceptance for MGChSP based on MOEL distribution

			$\frac{\mu}{\mu_0}$						
β	g	a	**1**	**2**	**4**	**6**	**8**	**10**	**12**
0.25	1	0.7	0.0824	0.2870	0.5474	0.6751	0.7478	0.7942	0.8264
	1	0.8	0.0591	0.2386	0.4997	0.6362	0.7159	0.7674	0.8033
	1	1.0	0.0315	0.1658	0.4156	0.5642	0.6554	0.7159	0.7586
	1	1.2	0.0176	0.1163	0.3453	0.4997	0.5993	0.6672	0.7159
	1	1.5	0.0080	0.0697	0.2617	0.4156	0.5230	0.5993	0.6554

	1	2.0	0.0025	0.0315	0.1658	0.3052	0.4156	0.4997	0.5642
0.10	1	0.7	0.0824	0.2870	0.5474	0.6751	0.7478	0.7942	0.8264
	1	0.8	0.0591	0.2386	0.4997	0.6362	0.7159	0.7674	0.8033
	1	1.0	0.0315	0.1658	0.4156	0.5642	0.6554	0.7159	0.7586
	1	1.2	0.0176	0.1163	0.3453	0.4997	0.5993	0.6672	0.7159
	1	1.5	0.0080	0.0697	0.2617	0.4156	0.5230	0.5993	0.6554
	1	2.0	0.0025	0.0315	0.1658	0.3052	0.4156	0.4997	0.5642
0.05	2	0.7	0.0027	0.0543	0.2553	0.4185	0.5303	0.6085	0.6652
	2	0.8	0.0012	0.0349	0.2054	0.3643	0.4797	0.5628	0.6243
	1	1.0	0.0315	0.1658	0.4156	0.5642	0.6554	0.7159	0.7586
	1	1.2	0.0176	0.1163	0.3453	0.4997	0.5993	0.6672	0.7159
	1	1.5	0.0080	0.0697	0.2617	0.4156	0.5230	0.5993	0.6554
	1	2.0	0.0025	0.0315	0.1658	0.3052	0.4156	0.4997	0.5642
0.01	2	0.7	0.0027	0.0543	0.2553	0.4185	0.5303	0.6085	0.6652
	2	0.8	0.0012	0.0349	0.2054	0.3643	0.4797	0.5628	0.6243
	2	1.0	0.0003	0.0146	0.1322	0.2743	0.3906	0.4797	0.5482
	2	1.2	0.0001	0.0062	0.0847	0.2054	0.3164	0.4071	0.4797
	1	1.5	0.0080	0.0697	0.2617	0.4156	0.5230	0.5993	0.6554
	1	2.0	0.0025	0.0315	0.1658	0.3052	0.4156	0.4997	0.5642

By taking $\beta = 0.25$ and $a = 0.7$ as control values from Tables 3, we can observe the $L(p)$ increases from 0.0824 to 0.8264 for MGChSP following MOEL distribution. In conclusion, an increase in mean ratio implies an increase in the probability of a lot of the product being accepted for the proposed plan since higher mean ratio represents higher quality of the product which has better chances of acceptance.

6. Conclusion

In this article, the modified group chain sampling plan (MGChSP) has successfully been developed based on MOEL distribution. Simultaneous inspection on multiple products is implemented in the proposed plan, which has reduced the sample size during the inspection. Therefore, cost and time saving are expected since both are closely related to the sample size used. Besides that, truncated life test is also applied in this article in order to avoid time and cost wasting by stopping the inspection at pre-determined termination time.

Acknowledgments

This research work has been fully funded by the University Grant (S/O Code: 13413).

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